The Life Cycle of Firms and the Productivity Advantages of Large Cities

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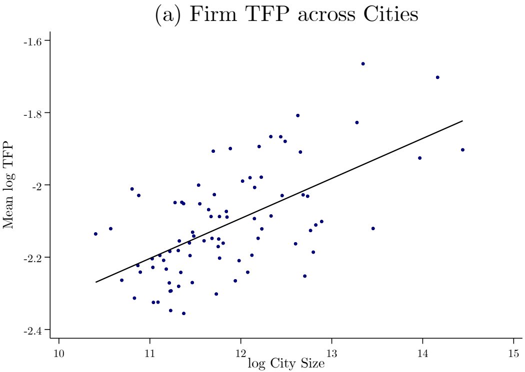
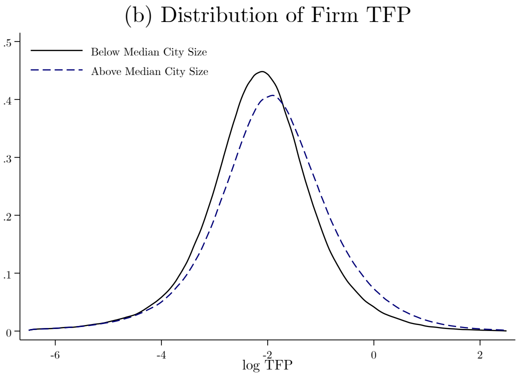
Macroeconomics Workshop

April 2024

Motivation

Firms are, on average, more productive in larger cities





Motivation

Firms are, on average, more productive in larger cities

- Origins of the urban productivity premium:

    1. Agglomeration economies (Rosenthal and Strange, 2004, Combes et al., 2012)

    2. Firm sorting (Behrens et al., 2014, Gaubert, 2018)

  → Relative strength of 1.–2. determines the potential effectiveness of place-based policies

- So far, quantification of 1.–2. has relied on static models

    – Firm dynamics and life-cycle growth are relevant for agg. productivity (Moll, 2014, Hsieh and Klenow, 2014)

  → **This paper**: use information on firm dynamics and firms life-cycle growth across cities to decompose the urban productivity premium into agglomeration and firm sorting

What I am doing

- Facts on firm dynamics (entry and exit) and life-cycle growth across the city-size distribution in Spain
  1. No relevant differences in firm entry and firm exit rates between large and small cities
  2. Firm growth over the life-cycle is higher in larger cities

- Canonical model of firm dynamics (Hopenhayn, 1992) augmented with
  + Agglomeration externality
  + Ex-ante (*productivity type*) and ex-post (*productivity shocks*) firm heterogeneity

  → higher firm growth in large cities may be due to
    - agglomeration forces
    - ex-ante high productivity firms slowly reaching their long-run size

What I am doing and what I find

- Model-based identification strategy: if
    - (a) ex-post productivity shocks are common across cities
    - (b) agglomeration economies are not firm age-dependent
        - $\Rightarrow$ Differences in firm growth across cities for old firms only depend on city-size, not on firm types
    - *Intuition:* old firms have reached their permanent productivity level, which is constant
      $\rightarrow$ their growth only depends on ex-post shocks and how these are amplified by city-size
- Calibration to match differences in firm growth and similarities in entry and exit rates across cities
- Finding: the urban productivity premium is mainly explained by firm sorting
    - Large complementarities between firm efficiency and city-size

Related Literature

1. Productivity advantages of large cities and firm sorting, selection, and agglomeration
   Rosenthal and Strange (2004), Combes, Duranton, Gobillon, Puga, and Roux (2012)
   Behrens, Duranton, and Robert-Nicoud (2014), Gaubert (2018), Ziv and Schoefer (2022)

   → A new identification strategy based on data and theory of firm dynamics

2. Firm dynamics across regions
   Brinkman, Coen-Pirani, and Siegel (2016), Walsh (2019), Brandt, Kambourov, and Storesletten (2019), Klenow and Li (2024)

   → A different question: revising why firms located in large cities are more productive

3. Firm growth over the life cycle
   Haltiwanger, Jarmin, Miranda (2013), Hsieh and Klenow (2014), Arkolakis (2016), Sterk, Sedláçek, Pugsley (2021), Kochen (2023)

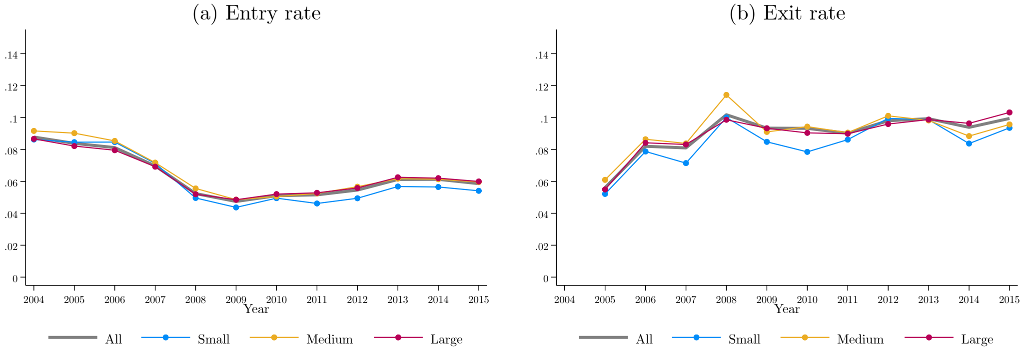
   → A look at its geographical dimension

Facts

Data

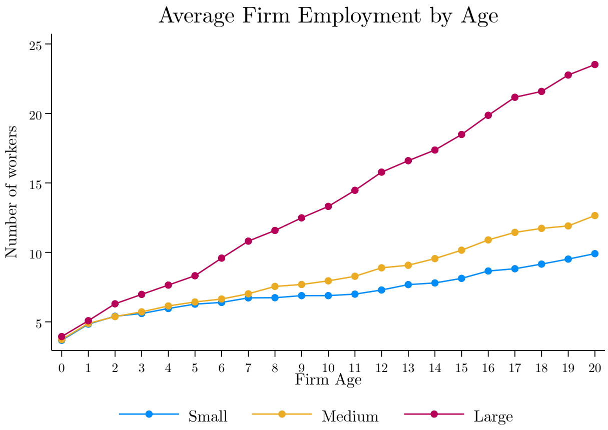
- Balance sheet information of non-financial Spanish firms, compiled by the Bank of Spain
  - Annual frequency 2004-2018, representative of Spanish market economy (Almunia et al., 2018)
  - Information on firm sector, employment, asset holdings, wage bill and **location** of headquarters
  - Drawbacks:
    - Data at the firm level rather than at the establishment-level
    - Imperfect to study firm exit, as no explicit reporting of firms ceasing operation
- Geography: 83 Urban Areas (UA) defined by Ministry of Transports and Mobility map
  - Notion of local labor market (68% of population, 73% of firms in full sample)
    - Final sample with 6 million firm-year observations, 913 thousand firms
  - Compute UA size as the number of people within 10km of the average person in UA (De la Roca & Puga, 2017)

1. Entry and exit rates are very similar for cities of different size



- **Entrants** are more productive in large cities, yet not larger SEE
- **Exiters** are more productive *and* larger in big cities SEE

2. Firm growth over the life cycle is higher in large cities



- Firms grow much more over their life-cycle in big cities
  - Holds within sector SEE
  - Also true in terms of value added SEE
- Driven by large firms becoming much larger in big cities, not by small firms exits
  - Firm survival rates invariant to city-size

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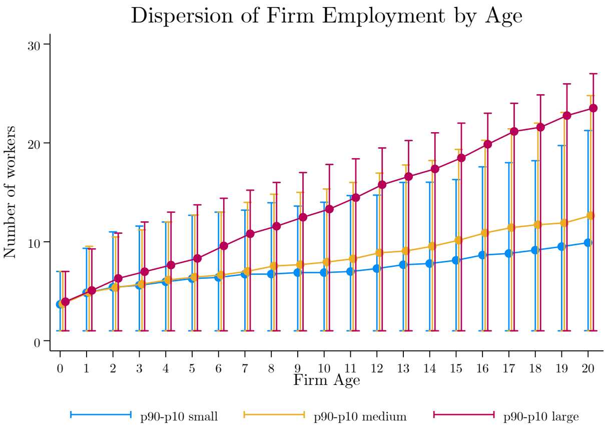

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From the Facts to a Quantitative Model

- In larger cities
  1. The entry and the exit rates are not different than in smaller cities
  2. Firms grow more over the life-cycle, both in terms of employment and value added

- A model that uses these facts to quantify the drivers of the urban productivity premium
  - Firm dynamics as in Hopenhayn (1992): endogenous entry and exit
  - Agglomeration externality: city size (population) increases firm TFP, more so for highly efficient firms
    - → Urban productivity premium due to agglomeration economies
  - Heterogeneous firms in long-run permanent efficiency
    - → Urban productivity premium due firm sorting

- The model is not a theory of firm location choices, but a measurement tool

Model

Environment

- A city economy (*or a world of isolated city-islands*), populated by:
  - Representative household of exogenous measure L (city-size)
    - Static problem, no savings
  - Heterogeneous firms of endogenous measure Ω
    - Heterogeneity in firm productivity $\varphi(z, L)$ due to idosyncratic firm efficiency z, which evolves stochastically
    - Firm productivity $\varphi(z, L)$ affected by city size L
    - Measure of firms Ω affected by endogenous firm entry and exit
- Stationary environment

Incumbent firms: production function

- Decreasing returns to scale technology that only uses labor ℓ, hired every period

$$y = \varphi(z, L)\ell^{\gamma} \quad \text{with} \quad \gamma < 1$$

Assumption 1. Firm productivity $\varphi(z, L)$ increases with firm idiosyncratic efficiency z and city-size L.

$$\frac{\partial \varphi(z, L)}{\partial z} > 0 \quad \text{and} \quad \frac{\partial \varphi(z, L)}{\partial L} > 0.$$

Assumption 2. Firm productivity $\varphi(z, L)$ is log-supermodular in firm idiosyncratic efficiency z and city-size L.

$$\frac{\partial^2 \log \varphi(z, L)}{\partial z \partial L} > 0.$$

$\rightarrow$ A1. captures standard agglomeration economies (e.g. labor pooling)

$\rightarrow$ A2. provides a rationale for firm sorting (e.g. spillovers in R&D)

Incumbent firms: dynamics

- Production entails fixed operating costs c_f paid in units of labor
  - At the start of every period, the firm may decide to exit the market and avoid paying c_f
  - Firm exit depends on the expected evolution of firm idiosyncratic efficiency z

Assumption 3. Firm idosyncratic efficiency z follows an exogenous process given by:

$$\log z_{i,a} = u_{i,a} + v_{i,a} + \varepsilon_{i,a},$$

$$u_{i,a} = \rho_u u_{i,a-1} + \theta_i, \qquad u_{i,-1} \sim iid(0, \sigma_u^2), \qquad \theta_i = iid(\mu_\theta, \sigma_\theta^2),$$

$$v_{i,a} = \rho_v v_{i,a-1} + \iota_{i,a}, \qquad v_{i,-1} = 0, \qquad \iota_{i,a} = iid(0, \sigma_\iota^2),$$

$$\varepsilon_{i,a} \sim iid(0, \sigma_\varepsilon^2),$$

where $z_{i,a}$ denotes the efficiency of firm i at age a

→ A3. $\log z_{i,a}$ combines “ex-ante” differences in firm efficiency with “ex-post” shocks to firm efficiency

Incumbent optimization and Firm entry

Optimization

- Static production problem $\rightarrow \ell(z, L, w) = \frac{\gamma}{w} \frac{\varphi(z,L)}{c(w)}^{\frac{1}{1-\gamma}}$

  $\rightarrow$ Firm growth $\Delta \log \ell(z, L, w)$ driven by the evolution of firm idiosyncratic z and by $\frac{\partial^2 \log \varphi(z,L)}{\partial z \partial L}$

- Dynamic exit decisions

$$V(v_{i,a}, a, u_{i,-1}, \theta_i) = \pi(z_{i,a}, L, w) - c_f \; + \; (1-\delta)\,\beta \max\left\{\mathbb{E}\left[V(v_{i,a+1}, a+1, u_{i,-1}, \theta_i)|v_{i,a}\right], 0\right\}$$

Firm entry

- After paying entry cost c_e in units of labor, entrants observe permanent efficiency θ_i and $u_{i,-1}$
  - May decide to operate and pay c_f or exit immediately and never produce

$\rightarrow$ Free entry condition $\int_{\theta,u_{-1}} \int_v V(v_0, 0, u_{-1}, \theta) dG(v, 0, u_{-1}, \theta) = wc_e$

Calibration

Calibration strategy

- Simulate three economies of the relative size L of small, medium and large cities
  - Firm productivity: $\log \varphi(z, L) = \alpha \log L + \log z \times (1 + \log L)^{\eta}$
    - $\alpha > 0 \rightarrow$ standard agglomeration parameter (A1.)
    - $\eta > 0 \rightarrow$ complementarity between firm efficiency z and city-size L, induces firm sorting (A2.)
    - City-size has no effect on firm productivity when $L = 1$, as $\log \varphi(z, 1) = \log z \rightarrow$ set $L = 1$ for small cities
  - Main targets:
    - Differences in firm life-cycle growth across cities
    - Similarities in exit rates across cities
  - Parameters:
    - Common across cities: entry costs c_e, operating costs c_f, ex-post shocks $\rho_v, \sigma_\iota, \sigma_\varepsilon$
    - City-specific: ex-ante heterogeneity $\mu_\theta, \sigma_\theta, \rho_u$

Identification I

- Firm employment level: $\log \ell_{i,a} = \log \gamma - \log w - \log c(w) + \alpha \log L + (1 + \log L)^{\eta} \log z_{i,a}$
  - Conditional on w, variation in $\log \ell_{i,a}$ across cities may be due to firm efficiency $\log z_{i,a}$ or city-size L
    - $\rightarrow$ Not useful to separately identify firm raw efficiency μ_θ from agglomeration forces α and η

- Firm employment growth: $\log \ell_{i,a+1} - \log \ell_{i,a} = (1 + \log L)^{\eta} (\log z_{i,a+1} - \log z_{i,a})$
  - Variation in $\Delta \log \ell_{i,a}$ across cities can separately identify η, particularly for old firms, as
    - Differences in $\Delta \log z_{i,a}$ across cites are only due to ex-ante efficiency, given ex-post shocks are common (A3.),
    - For old firms, ex-ante efficiency has converged to its constant long-run level, hence does not affect $\Delta \log z_{i,a}$
    - $\rightarrow$ Differences in $\Delta \log \ell_{i,a}$ across cities for old firms are only due to city-size L

- Identification conditional on firms remaining in operation $\rightarrow$ target firm exit rates by age

Identification II

- Firm employment level: $\log \ell_{i,a} = \log \gamma - \log w - \log c(w) + \alpha \log L + (1 + \log L)^{\eta} \log z_{i,a}$
  - Both α and μ_θ are associated with higher employment levels, yet
    - α affects employment levels of firms at any age in the same proportion, while μ_θ does not
    - μ_θ is a more important determinant of $\log z_{i,a}$ for old firms, as young firms still affected by initial shock u_{-1}

    $\rightarrow$ Higher μ_θ increases $\log z_{i,a}$ more for old firms, shifting rightwards the employment distribution by firm age

  $\rightarrow$ Targets:
    - Average employment level by age in each city
    - Distribution of employment by firm age in each city

Identification III

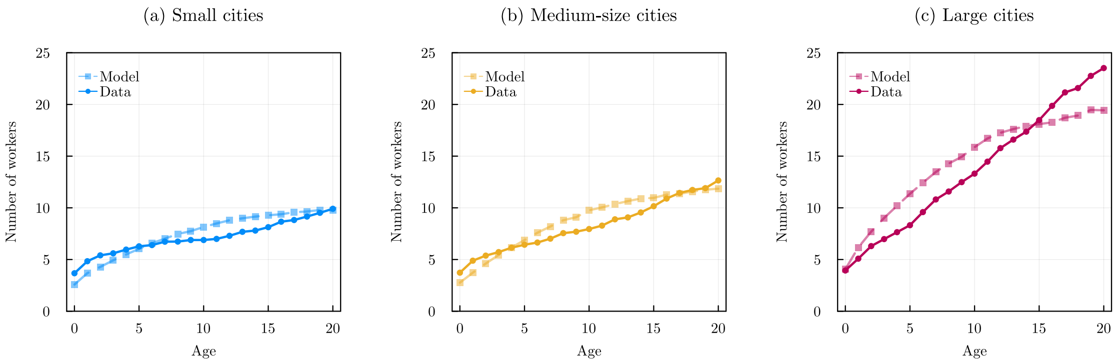
- Remaining parameters of the stochastic process for firm efficiency $z_{i,a}$: $\underbrace{\sigma_\theta, \rho_u, \sigma_u}_{\text{ex-ante het}}$ $\underbrace{\rho_v, \sigma_\iota \text{ and } \sigma_\varepsilon}_{\text{ex-post het}}$
  - Determine the autocovariance of firm-level $\log z_{i,a}$ $\rightarrow$ determine the autocovariance of firm-level $\log \ell_{i,a}$

$$\text{Cov}(\log \ell_{i,a}, \log \ell_{i,a-j}) = (1 + \log L)^{2\eta} \left[\sigma_\theta^2 \sum_{k=0}^{a-j} \rho_u^k \sum_{k=0}^{a} \rho_u^k + \sigma_u^2 \rho_u^{2(a+1)-j} + \sigma_\iota^2 \rho_v^j \sum_{k=0}^{a-j} \rho_v^{2k} + \mathbf{1}_{j=0}\, \sigma_\varepsilon^2 \right].$$

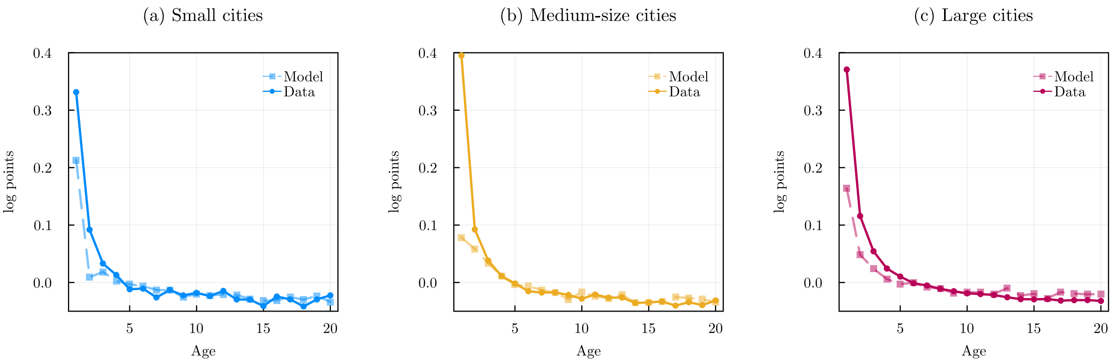
    - At short lags, both ex-ante and ex-post heterogeneity matter; at long lags, ex-post dominates
    - *Intuition:* if firm-level employment is only determined by ex-post shocks, no autocorrelation in the long-run

$\rightarrow$ Target: autocovariance of firm-level employment in each city (for a balanced panel of firms)

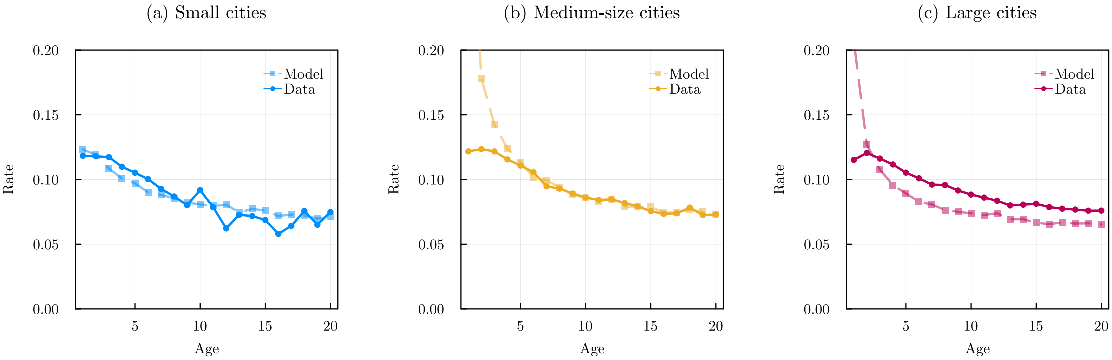
Model fit: average firm size



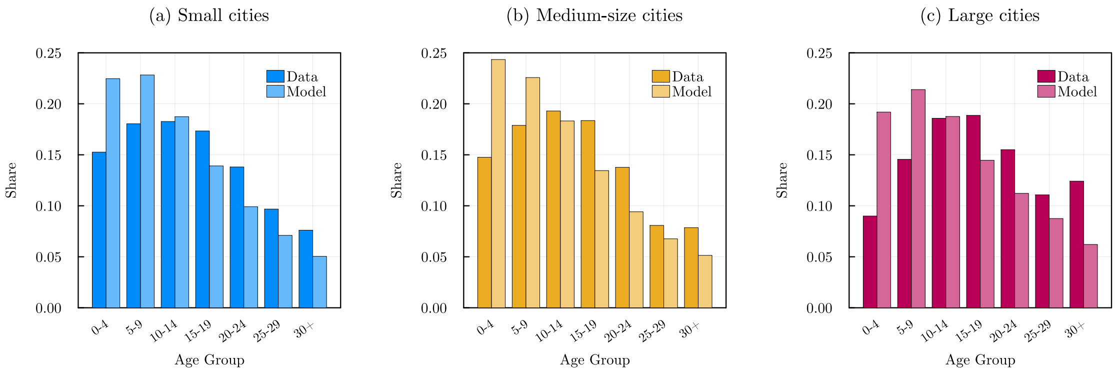
Model fit: average firm growth



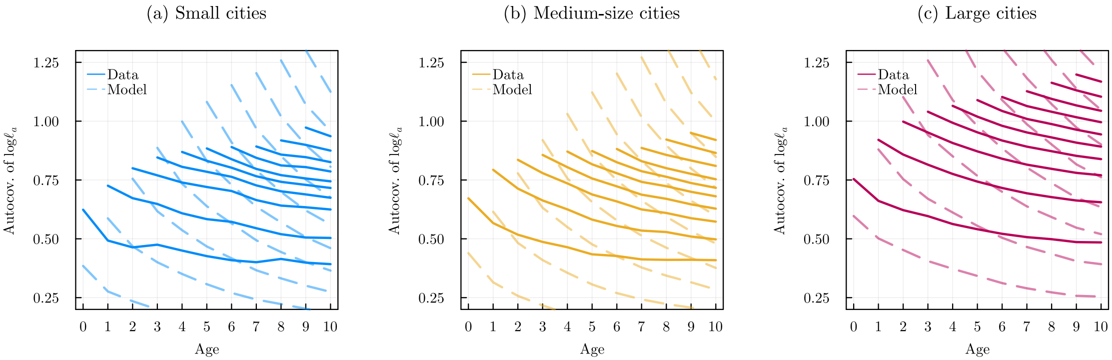
Model fit: exit rates by age



Model fit: employment distribution by age



Model fit: autocovariance of firm employment



Calibrated parameters

Common parameters

Parameter	Description	Value
Set a priori		
β	Discount rate	0.96
γ	Decreasing returns	0.8
Calibrated		
c_f	Fixed cost	1.539
c_e	Entry cost	0.517
δ	Exogenous exit rate	0.069
α	Agglomeration elasticity	0.03
η	Complementarity L and $z_{i,a}$	0.046
ρ_v	Persistence of ex-post shock	0.913
σ_ι	Std. dev. of idiosyncratic shock	0.596
σ_ϵ	Std. dev. of noise shock	0.191

Calibrated parameters

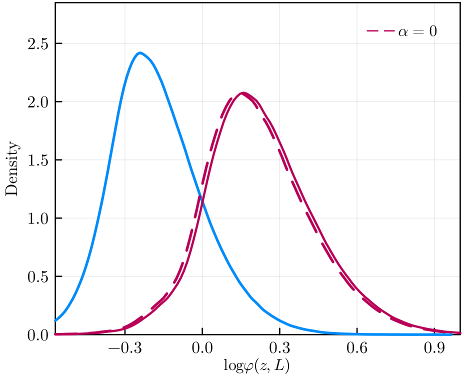
City-specific parameters

Parameter	Description	Value		
		Small	Medium	Large
μ_θ	Mean of ex-ante efficiency	-3.907	-3.421	-2.839
σ_θ	Std. dev. of ex-ante efficiency	1.043	1.076	1.471
ρ_u	Persistence initial distance shock	-0.106	0.086	0.135
σ_u	Std. dev. initial distance shock	1.017	0.73	0.787

The Origins of the Urban Productivity Premium

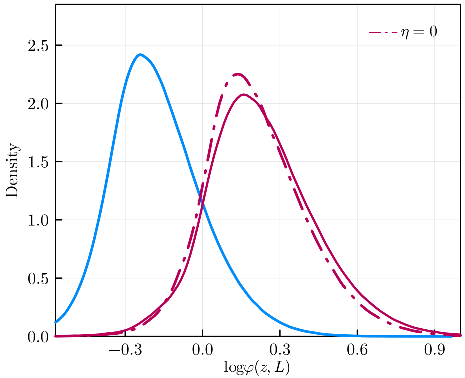
Decomposition of firm productivity

- Firm productivity: $\log \varphi(z, L) = \alpha \log L + \log z \times (1 + \log L)^{\eta}$
  - Exercise: set $\alpha = 0$
    - $\rightarrow$ Informative about the strength of agglomeration forces affecting all firms



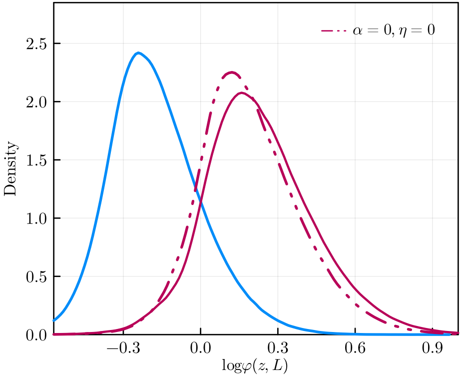
Decomposition of firm productivity

- Firm productivity: $\log\varphi(z,L) = \alpha\log L + \log z \times (1+\log L)^{\eta}$
  - Exercise: set $\eta = 0$
    - $\rightarrow$ Informative about the strength of the complementarity between city-size and firm efficiency



Decomposition of firm productivity

- Firm productivity: $\log \varphi(z, L) = \alpha \log L + \log z \times (1 + \log L)^{\eta}$
  - Exercise: set both $\alpha = 0$ and $\eta = 0$
    - $\rightarrow$ Informative about the productivity advantages of large cities



Decomposition of firm productivity

- Elasticity of firm productivity φ to city-size L

$$\log \varphi_i = \beta_0 + \beta_1 \log L + \varepsilon_i$$

	Benchmark	$\alpha = 0$	$\eta = 0$	$\alpha = 0, \eta = 0$	Same θ	Same $\theta, \eta = 0$
$\hat{\beta}_1$	0.129	0.124	0.121	0.116	0.001	0.003

$\rightarrow$ Urban productivity premium is mainly driven by the sorting of high-efficiency firms into large cities

Final remarks

- Firms located in large cities are more productive, what does this reflect?
  - Urban agglomeration economies
  - Sorting of highly efficient firms

- Using
  - information on firm dynamics and firm life-cycle growth across the city-size distribution in Spain
  - quantitative model of firm dynamics with agglomeration externalities and ex-ante firm heterogeneity

  $\rightarrow$ urban productivity premium is mainly due to firm sorting

- Implications for place-based policies

Appendix

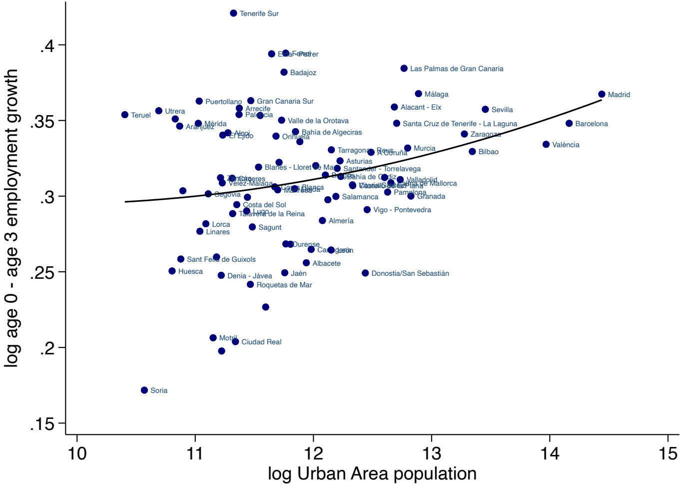
Urban Areas in Spain

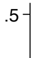
- Spain has 85 Urban Areas defined by the Ministry of Transports and Mobility

  Smallest is Teruel with 32,500 people in 2004; largest is Madrid with 5,472,387 people in 2004

BACK

Firm growth across the city-size distribution





Firm growth over the life cycle is higher in large cities

- Allow local population to have a different effect along the firm life cycle

$$\text{log firm growth}_{iust} = \alpha_{st} + \sum_{a}^{A} \gamma_a \mathbf{1}_{\{\text{Age}_{iust}=a\}} + \sum_{a}^{A} \beta_a \,\text{log population}_{ut} \times \mathbf{1}_{\{\text{Age}_{iust}=a\}} + \epsilon_{iust}$$

	log firm growth
Age=1 $\times$ log population	0.0009
Age=2 $\times$ log population	0.0107***
Age=3 $\times$ log population	0.0083***
Age=4 $\times$ log population	0.0050***
Age=5 $\times$ log population	0.0057***
Age=6 $\times$ log population	0.0045***
Age=7 $\times$ log population	0.0046***
Age=8 $\times$ log population	0.0012
Age=9 $\times$ log population	0.0023***
Age=10 $\times$ log population	0.0021***
Age=11 $\times$ log population	0.0019**
Age=12 $\times$ log population	0.0011
Age=13 $\times$ log population	0.0004
Age=14 $\times$ log population	0.0005
Age=15 $\times$ log population	0.0004
Observations	4232072
R^2	0.051

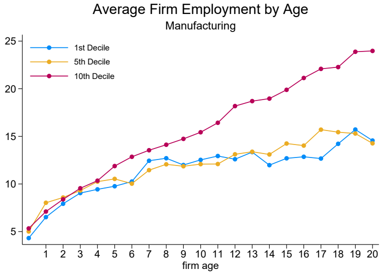
$^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

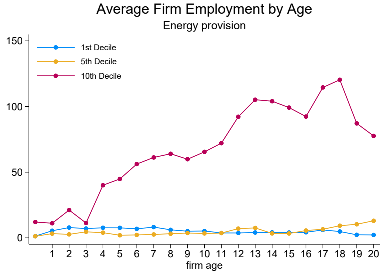
→ Firms grow more in large cities over their life-cycle (controlling by sector and age)

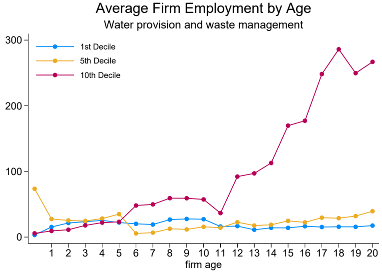
BACK

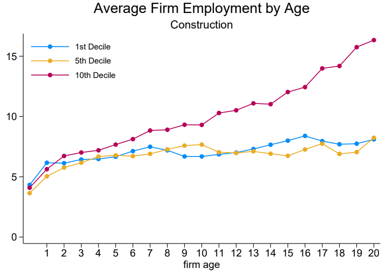
Firm growth over the life cycle is higher in large cities

BACK

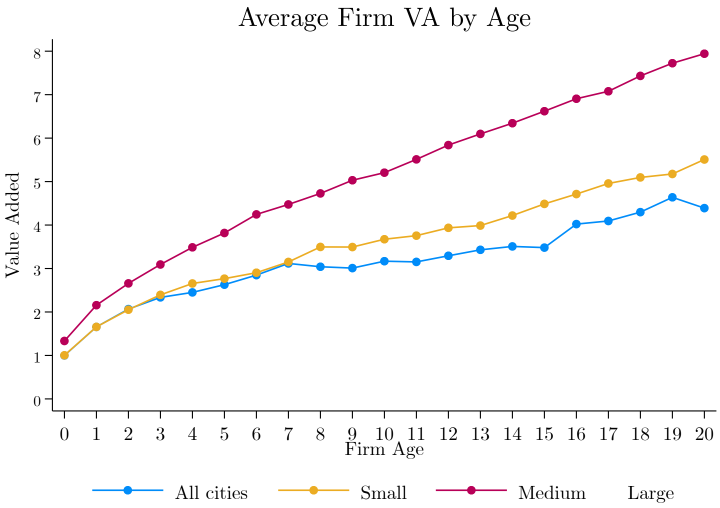








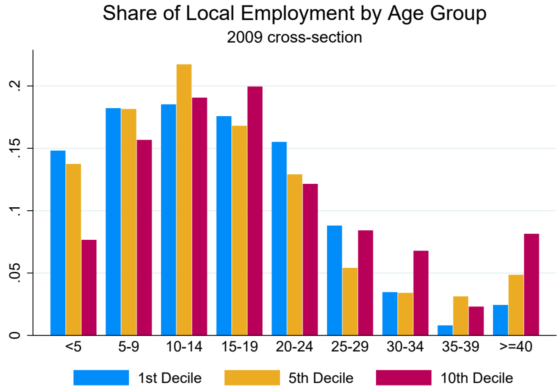
VA growth over the life cycle is higher in large cities

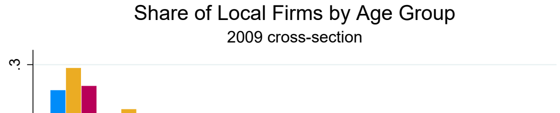


BACK

Firm growth over the life cycle is higher in large cities

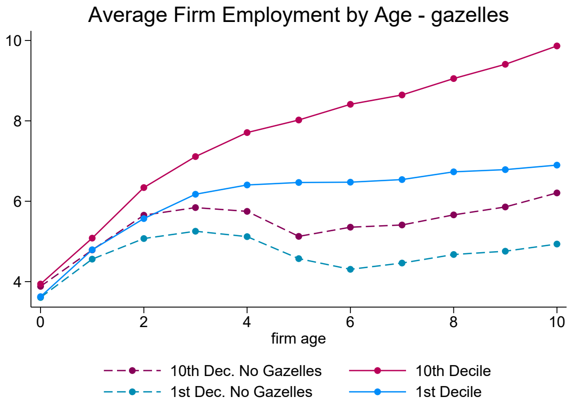
Corollary





Higher firm growth in large cities is driven by small group of high-growth firms

- The literature has emphasized that a small group of young firms (*gazelles*) account for a large share of employment growth (Haltiwanger et al. (2016), Sterk et al. (2021)) → define gazelles as firms that
  (a) Grow at an annualized rate of 20% for their first 5 years of operation
  (b) Reach at least 10 employees at some point during their life-cycle



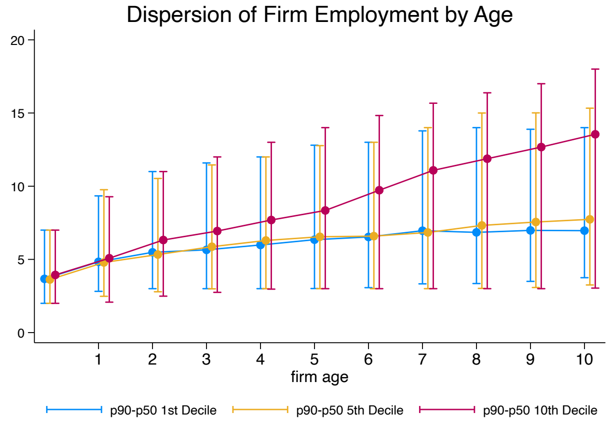
- Only 4.1% of all startups in the economy
  - In smallest cities 3.9%, in largest 4.3%
  - Similar exit profiles across cities

  → Gazelles manage to scale up in large cities

BACK

Large firms drive the higher average life cycle growth in cities

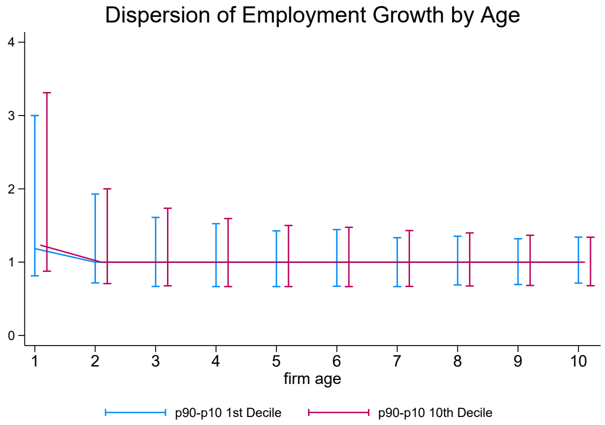
- Compute the 90th and 50th percentile of the employment distribution at each age, for each UA size-decile



BACK

Dispersion of firm growth is higher in large cities

- Compute the 90th and 10th percentile of the employment growth distribution at each age, for each UA size-decile



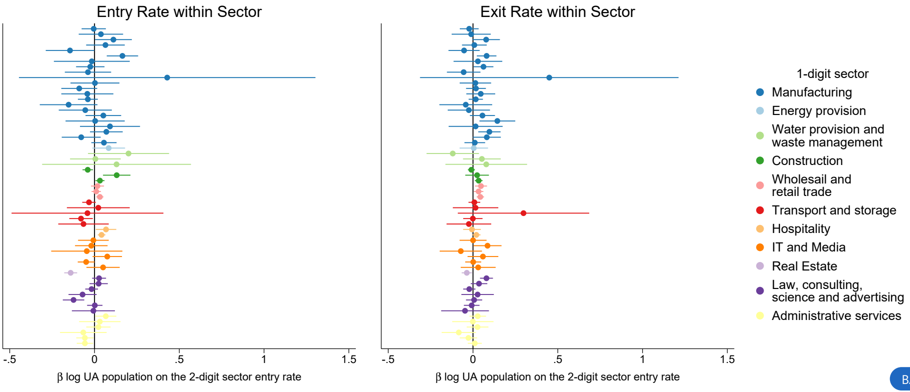
BACK

Entry and exit rates (at the sector level) are similar for cities of different size

- Some sectors may be characterized by higher entry and exit rates, and may be differenty present in large and small cities

  → Define a market as UA – 2-digit-sector combination and compute entry and exit rates as before

  - Run Poisson regression of entry/exit rate on city-size for each sector
    $y_{ust} = \exp\{\alpha_t + \beta \log \text{population}_{ut} + \epsilon_{ust}\}$



BACK

Entrants in larger cities are more productive, yet not larger

- Regress firm K, L and TFPQ on city size, controlling by year or year-sector FE

	log TFPQ	log TFPQ	log K	log K	log L	log L
log population	0.0649***	0.0529***	-0.0171***	0.0094***	-0.0211***	-0.0045***
	(0.0025)	(0.0024)	(0.0036)	(0.0035)	(0.0014)	(0.0013)
Year FE	Yes	–	Yes	–	Yes	–
2-dig sector–year FE	No	Yes	No	Yes	No	Yes
Observations	215740	215726	250059	250047	329755	329746
R^2	0.006	0.084	0.006	0.101	0.004	0.082

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

BACK

Exiters in larger cities are larger and more productive

- Regress firm K, L and TFPQ on city size, controlling by year or year-sector FE

	log TFPQ	log TFPQ	log K	log K	log L	log L
log population	0.0960***	0.0870***	0.0030	0.0080***	-0.0035***	0.0120***
	(0.0017)	(0.0017)	(0.0028)	(0.0027)	(0.0012)	(0.0011)
Year FE	Yes	–	Yes	–	Yes	–
2-dig sector–year FE	No	Yes	No	Yes	No	Yes
Observations	510486	510475	603936	603926	723037	723030
R^2	0.028	0.111	0.007	0.108	0.027	0.117

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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